PFC/JA-90-2

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January 1990

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Submitted for Publication in *The Physics of Fluids B*

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Abstract The power reflection coefficient for D(H), DT(^{3}He), and D(^{3}He) ion cyclotron heating is found in closed form. The fast wave equation previously solved numerically is solved by a slowly varying amplitude expansion about the Budden equation asymptotics. The coupling between the fast Alfvén wave and the ion Bernstein wave is described by this formalism. The dissipation associated with the ion cyclotron resonance is accounted for by the slow variation. Validity criteria for this method are presented.

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I. INTRODUCTION

In ICRF heating a fast Alfvén wave (FAW) is excited at the edge of the plasma. It propagates in and encounters a resonance and a singular layer where it can partially transmit, convert to a kinetic ion Bernstein wave (IBW), reflect, and dissipate some energy into the plasma. This layer is termed a mode conversion region. Geometrical optics is not valid in this region where abrupt changes of the mode structure occurs. In general the problem can be formulated as an integro-differential or approximate, high-order differential equation and solved numerically[1−5]. However, in order to make predictions and to better understand the scaling laws, closed form solutions of the scattering coefficients are desirable.

Currently there has been an effort to 'order-reduce' the problem. The fourth or higher order differential equations necessary to solve the problem are reduced to combinations of lower order equations. By this technique, a first order differential equation has been found that describes the transmission properties[6], and a set of coupled equations that can be solved analytically has been found to give the conversion and transmission coefficients including the effects of dissipation[7]. An equation has also been found to model the reflection and transmission of the FAW[8]. Thus far this second order differential equation known as the 'fast wave approximation' has been solved numerically to find the reflection coefficient. The results of this equation have been shown to agree well with the results of other investigators[9]. In this paper we find an approximate closed form solution for the reflection coefficient. We thus, finally have closed form solutions for all of the scattering coefficients using this order reduction technique. By an entirely different technique, Ye and Kaufman have also produced an analytic solution for this problem[10]. Their published results are for the pure second harmonic case and they have shown it to compare well with other investigators. Their results for minority heating have not yet been published.

II. THE FAST WAVE EQUATION

We solve the fast wave equation given by

$$\Phi_{\xi\xi} + q^2(\xi)\Phi = 0, \tag{1}$$

where

$$q^2(\xi) = N_c^2\Big[1 - \Big(\frac{\lambda_N}{2\tau+\lambda_N}\Big)\frac{\Gamma}{-\tau+\Gamma}\Big], \tag{2}$$

$$N_c^2 = \lambda_N\frac{(2\tau+\lambda_N)}{\tau}, \tag{3}$$

$$\Gamma = R_A\sum_{i=1}^{M}\frac{b_i}{a_i}Z\Big(\frac{\xi}{a_i}\Big), \tag{4}$$

$$\xi = x\frac{\omega}{c_A}, \quad R_A = R_0\frac{\omega}{c_A}, \quad c_A^2 = \frac{B_0^2}{\mu_0 n_1 m_1}, \tag{5}$$

R_0 is the major radius, c_A is the majority Alfvén speed, M is the number of resonant species. The parameters τ, λ_N, a_i, b_i are particular to a given heating scenerio and are derived from the corresponding dispersion relation. Section III will give examples for D(H), D(^{3}He) and DT(^{3}He) heating. Equation (1) is precisely 'the fast wave approximation' derived originally for D(H) heating[8] and later generalized to other heating scenerios[9]. To unify the treatment for various cases, we have changed the notation. The connection between this notation and the previous, as well as a derivation of the fast wave equation is given in the appendix.

Figure 1 gives a sketch of the real and imaginary parts of $q^2(\xi)$. The real part looks very much like a resonance broadened 'Budden' potential. The imaginary part has two peaks for certain parameter regimes which will be specified later. The left most peak corresponds to the 'Budden' like potential. The other peak centered about $\xi = 0$ corresponds to the ion cyclotron resonance. For D(H) heating this will be the minority fundamental which is degenerate with the majority second harmonic. For DT(^{3}He) heating the minority fundamental is degenerate with the tritium (if present) second harmonic. The 'Budden' part includes the effects of mode conversion to the ion Bernstein wave and any dissipation that may be present in that particular process. Therefore, to solve for the scattering coefficients R and T of the fast wave equation (1), we approximate the potential $q^2(\xi)$ with a Budden potential, for which analytic solutions exist[11]. The ion cyclotron part is then treated as a perturbation which effects a slowly varying amplitude on the asymptotic Budden solutions.

We consider the cases for large and small argument of the Z function. Firstly, take the large argument limit using the large argument expansion of the Z function in the potential. This corresponds to a position far from the ion cyclotron resonance layer. This yields

$$q^2(\xi) = N_c^2\Big[1 - \Big(\frac{\lambda_N}{2\tau + \lambda_N}\Big)\frac{-\gamma + i\sqrt{\pi}\xi\epsilon}{(-\tau\xi - \gamma) + i\sqrt{\pi}\xi\epsilon}\Big], \tag{6}$$

where

$$\gamma = R_A \sum_i b_i, \tag{7}$$

$$\epsilon = R_A \sum_i \frac{b_i}{a_i} e^{-\xi^2/a_i^2}. \tag{8}$$

Hence

$$\mathrm{Re}\, q^2(\xi) = N_c^2\Big[1 - \Big(\frac{\lambda_N}{2\tau + \lambda_N}\Big)\frac{\gamma(\tau\xi + \gamma) + \pi\xi^2\epsilon^2}{(\tau\xi + \gamma)^2 + \pi\xi^2\epsilon^2}\Big], \tag{9}$$

$$\mathrm{Im}\, q^2(\xi) = \sqrt{\pi}\lambda_N^2 \frac{\epsilon\xi^2}{(\tau\xi + \gamma)^2 + \pi\xi^2\epsilon^2}, \tag{10}$$

where (3) has been used to simplify the expression.

Now consider the small argument limit to obtain the behavior near the ion cyclotron resonance. In this case

$$q^2(\xi) = N_c^2\Big[1 - \Big(\frac{\lambda_N}{2\tau + \lambda_N}\Big)\frac{-\alpha\xi + i\sqrt{\pi}\epsilon}{(-\tau - \alpha\xi) + i\sqrt{\pi}\epsilon}\Big], \tag{11}$$

where

$$\alpha = 2R_A \sum_i \frac{b_i}{a_i^2}. \tag{12}$$

Hence

$$\mathrm{Re}\, q^2(\xi) = N_c^2\Big[1 - \Big(\frac{\lambda_N}{2\tau + \lambda_N}\Big)\frac{\alpha\xi(\tau - \alpha\xi) + \pi\epsilon^2}{(\tau + \alpha\xi)^2 + \pi\epsilon^2}\Big], \tag{13}$$

$$\mathrm{Im}\, q^2(\xi) = \sqrt{\pi}\lambda_N^2 \frac{\tau\epsilon}{(\tau + \alpha\xi)^2 + \pi\epsilon^2}. \tag{14}$$

From expressions (9) and (13) we see that the real part of the potential is well approximated by the large argument expansion form even in the regions where only the small argument expansion applies.

The large argument form of $q^2(\xi)$ can be further simplified. The second term in the denominator of the imaginary part (10) is always small and will be insignificant except when $\xi \equiv \xi_r \simeq -\gamma/\tau$, which is the location of the 'Budden' resonance. Thus we substitute this value of ξ into that term and into the numerator turning (10) into a Lorentzian. We then replace the large argument form of $q^2(\xi)$ (6) with

$$q^2(\xi) = N_c^2\Big[1 + \Big(\frac{\lambda_N}{2\tau + \lambda_N}\Big)\frac{\gamma}{(-\tau\xi - \gamma) + i\sqrt{\pi}\xi_r\epsilon_r}\Big], \tag{15}$$

where ϵ_r is ϵ evaluated at the resonance ξ_r. One can verify that this gives the simplified Lorentzian form of the imaginary part and well approximates the real part. This form (15) of $q^2(\xi)$ has the form of a complex Budden potential for which an analytic solution is known. The ion cyclotron resonance is then treated as a perturbation on this potential. The criteria for this procedure to be valid is given in section II. The fast wave equation (1) becomes

$$\Phi_{\xi\xi} + N_c^2\Big[1 + \Big(\frac{\lambda_N}{2\tau + \lambda_N}\Big)\frac{\gamma}{(-\tau\xi - \gamma) + i\sqrt{\pi}\xi_r\epsilon_r} + if(\xi)\Big]\Phi = 0, \tag{16}$$

where the ion cyclotron perturbation is represented by $if(\xi)N_c^2$. We will find later that only the integral over $f(\xi)$ is important, rather than the specific form. We make the substitution $s = N_c(-\tau\xi - \gamma + i\sqrt{\pi}\xi_r\epsilon_r)/\tau$ which yields

$$\Phi_{ss} + \big[1 + \frac{s_0}{s} + ig(s)\big]\Phi = 0, \tag{17}$$

where

$$s_0 = \frac{N_c^3}{(2\tau + \lambda_N)^2}\gamma, \tag{18}$$

and $g(s)$ is $N_c^2 f(\xi)$ in the transformed coordinate. Note that s is complex and that s_0 is precisely the tunneling length for the transmission coefficient derived previously by various means[6–8]. Equation (17) without the perturbation is the complex Budden equation with asymptotic solutions, $|s| \to \infty$, given by White and Chen[11]. We let the solution of (17) be the product of the unperturbed solution $\Psi(s)$ and that of a slowly varying potential $A(s)$ so that $\Phi(s) = A(s)\Psi(s)$. We substitute this into (17), make the assumption $A_{ss} \ll gA$ and obtain

$$2A_s\Psi_s + ig(s)\Psi A \simeq 0. \tag{19}$$

From the White and Chen solution for $|s| \gg 1$

$$\frac{\Psi_s}{\Psi} \sim \pm i, \tag{20}$$

the + and − signs refering to outgoing and ingoing waves respectively. Thus we find

$$A \sim \exp\Big(\pm \int^{s} \frac{g(s')}{2} ds'\Big), \tag{21}$$

and furthermore

$$T = e^{-\pi s_0} e^{-2\mu}, \tag{22}$$

$$R = e^{-4\mu} e^{-4\nu} (1 - T^2)^2, \tag{23}$$

where

$$\nu = \sqrt{\pi} \frac{N_c}{\tau^2} \gamma \epsilon_r, \tag{24}$$

$$\mu = \Big| \int_0^\infty \frac{g(s)}{2} ds \Big|. \tag{25}$$

μ comes from the cyclotron damping. The transmission coefficient has an extra factor of $\exp(-2\mu)$ as compared with previous derivations[6–8]. We will see in section III that s_0 always dominates μ agreeing with the previous results. As mentioned, the exact form of the IC resonance perturbation is not required but we must approximate its integral. We do so with a simple approximation treating the IC resonance as a Lorentzian. We take the small argument form (14), let $\epsilon = \epsilon_0$ (ϵ evaluated at $\xi = 0$), and integrate over the result to yield

$$\mu \simeq \frac{\pi}{2} \frac{\lambda_N^2}{\alpha N_c}. \tag{26}$$

III. VALIDITY CRITERIA FOR THE APPROXIMATIONS

For the solution to be valid, two criteria must be satisfied. The first follows from the condition that the ion cyclotron resonance and the 'Budden' resonance be well separated. This is essential so that the ion cyclotron resonance can be treated as a perturbation on the asymptotic solutions of the complex Budden equation. This will set a limit on the maximum $N_{||}$ allowed. The half width of the ion cyclotron resonance is given approximately by $\xi = a_m$, where $a_m = \max\{a_i\}$ (necessary when two species are resonant). We thus set this value equal to the position of the 'Budden' resonance for the first criteria. This yields

$$N_{\|} < \frac{1}{\tau\sqrt{\beta_m}} \sum_i b_i. \tag{27}$$

The second criterium should imply a slowly varying amplitude, $A_{ss} \ll gA$, which with (21) gives

$$\left|\frac{1}{2g}\left(\frac{dg}{ds} + \frac{g}{4}\right)\right| \ll 1. \tag{28}$$

We will require both terms to be much less than unity. The maximum of g is given by the peak of (14) and with the proper normalization it is

$$g \equiv g_0 \simeq \frac{\lambda_N^2}{\sqrt{\pi} N_c^2 \epsilon_0}, \tag{29}$$

where ϵ_0 is ϵ evaluated at $\xi = 0$ An estimation of $|(dg/ds)/2g|$ is not as straightforward. For real argument of the Z function the imaginary part of $q(\xi)^2$ can be written as

$$\mathrm{Im}\, q^2(\xi) = \sqrt{\pi}\lambda_N^2 \frac{\epsilon}{(-\tau + \mathrm{Re}\,\Gamma)^2 + \pi\epsilon^2}. \tag{30}$$

We observe that (30) will approach zero slower than (14) as $|\xi|$ gets large since $\mathrm{Re}\,\Gamma$ is bounded and always increases at a rate slower than linear. Thus we can use (14) to obtain an upper bound for the slope of $g(s)$. If we let $\epsilon = \epsilon_0$ than (14) is a simple Lorentzian with an inflection point at $(\tau + \alpha\xi) = \sqrt{\pi}\epsilon_0 \,/\sqrt{3}$. This simple approximation is good enough to give an order of magnitude estimation for the validity criteria. We use this approximate form for g to obtain

$$\left|\frac{1}{2g}\frac{dg}{ds}\right| \simeq \frac{\alpha(\tau + \alpha\xi)}{(\tau + \alpha\xi)^2 + \pi\epsilon_0^2}. \tag{31}$$

Near the inflection point (31) is largest and is given by

$$\left|\frac{1}{2g}\left(\frac{dg}{ds}\right)\right|_0 \simeq \frac{\alpha}{\epsilon_0}. \tag{32}$$

IV. EXAMPLES

We calculate results for test cases involving DT(^{3}He) for CIT, D(^{3}He) for Alcator C-Mod, and D(H) for PLT. The parameters for the fast wave equation (1) are derived from the zero-electron-mass Vlasov-Maxwell local dispersion relation where the dielectric tensor elements have been expanded to first order in $(k_\perp \rho_i)^2$ (see the Appendix).

DT(^{3}He) and D(^{3}He)

For these cases the parameters of the fast wave equation (1) are:

$$\lambda_N = \frac{3}{7} + \frac{3}{4}\eta + \frac{\theta}{2} - N_\parallel^2, \tag{33}$$

$$\tau = \frac{9}{7} + \frac{\theta}{2} + \frac{3}{8}\eta + N_\parallel^2, \tag{34}$$

$$b_1 = \frac{3}{4}\eta + \frac{39}{40}\eta\beta_{He}N_c^2, \qquad a_1 = N_\parallel\sqrt{\beta_{He}}R_A, \tag{35}$$

$$b_2 = \frac{3}{8}\theta\beta_T N_c^2, \qquad a_2 = N_\parallel\sqrt{\beta_T}R_A. \tag{36}$$

where

$$\eta = \frac{n_{He}}{n_D}, \quad \theta = \frac{n_T}{n_D}, \quad \beta_i = \frac{2T_i}{m_i c_A^2}, \quad N_\parallel = k_\parallel\frac{c_A}{\omega}, \tag{37}$$

where c_A is the D Alfvén speed. The reflection and transmission coefficients are then given by expressions (22) and (23) with the following definitions:

$$\mu = \frac{\pi\lambda_N^2\beta_{He}R_A N_\parallel^2}{4N_c\Sigma}, \tag{38}$$

$$\nu = \frac{\sqrt{\pi}N_c R_A\Sigma^2}{\tau^2\sqrt{\beta_{He}}N_\parallel}\exp\left(-\frac{\Sigma^2}{\tau^2\beta_{He}N_\parallel^2}\right), \tag{39}$$

$$s_0 = \frac{N_c^3 R_A\Sigma}{(3 + 3\theta/2 + 3\eta/2 + N_\parallel^2)^2}, \tag{40}$$

where

$$\Sigma = \frac{3}{4}\eta + \frac{39}{40}\eta\beta_{He}N_c^2 + \frac{3}{8}\theta\beta_{He}N_c^2, \tag{41}$$

The ion temperatures have been set equal for simplicity. This would correspond to the early stages of heating. In the latter stages of heating this may not be true and the full expressions, keeping the temperatures separate should be used. The criterium (27) on $N_\parallel$ is

$$N_\parallel < \frac{1}{\tau\sqrt{\beta_{He}}}\Sigma, \tag{42}$$

This constraint (42) ensures that s_0 always dominates μ in the transmission coefficient (22).

The slowly varying criterium is given by (28) with

$$g_0 \simeq \frac{\lambda_N^2 \sqrt{\beta_{He}} N_\parallel}{\sqrt{\pi} N_c^2 \Sigma}, \tag{43}$$

$$\left| \frac{1}{2g} \left(\frac{dg}{ds} \right) \right|_0 \simeq \frac{2}{N_\parallel \sqrt{\beta_{He}} R_A}. \tag{44}$$

We observe that g_0 is always small because of the constraint on $N_\parallel$. For $N_\parallel < 2/R_A\sqrt{\beta_{He}}$, (44) is not small and the slowly varying criteria (28) is violated. However this does not invalidate the theory. As $N_\parallel$ approaches zero, both the width and the amplitude of $g(s)$ also go to zero. For very small $N_\parallel$, the perturbation due to $g(s)$ is negligible so the the full solutions to the fast wave equation are just the asymptotic Budden equation solutions. Only when the wavelength of the FAW and the halfwidth of $g(s)$ are of the same order does the perturbation become significant. Taking a_{He} to be the half width of $g(s)$ and $1/N_c$ to be the wavelength gives the condition $N_\parallel > 1/N_c\sqrt{\beta_{He}}R_A$. N_c is of order unity so we see that the slowly varying approximation becomes valid when the perturbation due to $g(s)$ becomes significant. In other words, for small $N_\parallel$ there is very little ion cyclotron damping so we can ignore it. When the damping becomes important it is manifested as a slowly varying amplitude. The validity of the theory is weakest during the transition between negligible damping to when the slowly varying approximation is applicable. However, the slowly varying criteria (28) is a local criteria while expression (44) is evaluated at the inflection point of $g(s)$ where the slope is at its maximum. The main contribution of $g(s)$ comes from near the peak where the slope is much flatter and thus the slowly varying criteria is satisfied better than what (44) would suggest.

We calculate two CIT test cases with DT(^{3}He) and an Alcator C-mod test case for D(^{3}He).

1. CIT DT(^{3}He) Case 1

The parameters are: $R_0 = 2.1$m, $B_0 = 10$T, $T_i = 20$KeV, $n_e = 6\times10^{20}\text{m}^{-3}$, $n_D = n_T$, $n_{He}/n_e = 0.05$, $f = 95$MHz. From the $N_\parallel$ condition (42) we find $N_\parallel < .4$ or $k_\parallel < 28\text{m}^{-1}$. Figure 2 shows the reflection coefficient found by solving the fast wave equation

(1) numerically and the closed form expression (23) plotted against $k_{\|}$. We see that there is a good agreement. From (40) we see that s_0 is large and always dominates μ within the range of allowable $N_{\|}$. For large s_0 the transmission coefficient is negligible so R has the simple form $R = \exp(-4(\mu + \nu))$. When T is very small it has been shown that the conversion coefficient T is also negligible so we have the situation where $D + R \sim 1$, where D is the coefficient of power dissipated. Therefore whatever is not reflected is damped. We can analytically determine where the reflection coefficient falls to a half and the damping begins to dominate. From (38) we find $\mu = 11N_{\|}^2$ We set $R = .5$ and solve for $N_{\|}$. In the range of relevent $N_{\|}$, μ will dominate ν so the equation is trivial to solve. This gives $N_{\|} \sim .12$ or $k_{\|} \sim 8\mathrm{m}^{-1}$ which corroborates with figure 2. T is small because $R_A\Sigma$ is very large. Therefore by choosing parameters such that $R_A\Sigma$ is made much larger than unity simplifies the scattering problem to that of reflection versus damping.

2. CIT DT(^{3}He) Case 2

The parameters are: $R_0 = 1.22\mathrm{m}$, $B_0 = 10\mathrm{T}$, $T_i = 10\mathrm{KeV}$, $n_D = n_T$, $n_e = 5 \times 10^{20}\mathrm{m}^{-3}$, $n_D = n_T$, $n_{He}/n_e = 0.05$, $f = 95\mathrm{MHz}$. The validity condition is $N_{\|} < .5$ or $k_{\|} < 33\mathrm{m}^{-1}$. The numerical and analytical reflection coefficients are plotted against $k_{\|}$ on figure 3. There is a fairly good agreement. We find s_0 is large and dominates μ. T is negligible and we can find where $R = 1/2$. The calculation gives $N_{\|} \simeq .23$ or $k_{\|} \simeq 15\mathrm{m}^{-1}$. For this $N_{\|}$, μ dominates ν.

3. Alcator C-mod D(^{3}He)

The parameters are: $R_0 = .64\mathrm{m}$, $B_0 = 9\mathrm{T}$, $T_i = 2\mathrm{KeV}$, $n_e = 5 \times 10^{20}\mathrm{m}^{-3}$, $n_{He}/n_e = 0.05$, $f = 80\mathrm{MHz}$. The validity condition is $N_{\|} < .5$ or $k_{\|} < 40\mathrm{m}^{-1}$. See figure 4 for the plot of the reflection coefficient. In this case the transmission coefficient is not negligible but s_0 still dominates μ. The analytical solution agrees with the numerical one.

D(H)

For a D(H) plasma we have

$$\lambda_N = \frac{1}{3} + \frac{\eta}{4} - N_{\|}^2, \tag{45}$$

$$\tau = \frac{1}{3} + \frac{\eta}{8} + N_{\|}^2, \tag{46}$$

$$b_1 = \frac{\beta_D}{4} N_c^2, \qquad a_1 = N_\| \sqrt{\beta_D} R_A, \tag{47}$$

$$b_2 = \frac{\eta}{4}, \qquad a_2 = N_\| \sqrt{\beta_H} R_A, \tag{48}$$

where

$$\eta = \frac{n_H}{n_D}, \quad \beta_i = 2\frac{T_i}{m_i c_A^2}, \quad N_\| = k_\| \frac{c_A}{\omega}. \tag{49}$$

These then give

$$s_0 = \frac{N_c^3 R_A(\beta_D N_c^2 + \eta)}{1 + 3\eta/8 + N_\|^2}, \tag{50}$$

$$\mu = \frac{\pi \lambda_N^2 R_A N_\|^2}{N_c(N_c^2 + \eta/\beta_H)}. \tag{51}$$

We note that β_H is larger than β_D by a factor of two for equal temperatures. The minority H will also heat faster than the majority D. Thus in expression (24) for ν we can neglect the first term in ϵ to obtain

$$\nu \simeq \frac{\sqrt{\pi} N_c^2 R_A(\beta_D N_c^2 + \eta)\eta}{16\tau^2\sqrt{\beta_{He}} N_\|} \exp\Big(-\frac{(\beta_D N_c^2 + \eta)^2}{\tau^2 \beta_{He} N_\|^2}\Big). \tag{52}$$

The $N_\|$ condition (27) is

$$N_\| < \frac{1}{\tau\sqrt{\beta_D}}\Big(\frac{\beta_D}{4} N_c^2 + \frac{\eta}{4}\Big). \tag{53}$$

This condition again ensures s_0 dominates μ in T (22).

The argument for the validity of the slowly varying criteria used for DT(^{3}He) and D(^{3}He) also applies to the case of D(H). We compute the reflection coefficient for a PLT test case with parameters: $R_0 = 3$m, $B_0 = 4$T, $T_i = 5$KeV, $n_e = 4 \times 10^{19}\text{m}^{-3}$, $n_H/n_e = 0.05$, $f = 60$MHz. This case requires $N_\| < .6$ or $k_\| < 23\text{m}^{-1}$. Figure 5 gives a plot of the numerical and analytical reflection coefficients versus $k_\|$. We find a reasonable agreement. The transmission coefficient for this range of $k_\|$ is small and s_0 dominates μ. We find that at $k_\| \sim 6\text{m}^{-1}$, $R \simeq 1/2$ and μ dominates ν here.

V. CONCLUSIONS

We have found an approximate closed form solution for the reflection coefficient by solving the fast wave approximation in a perturbation scheme where the ion cyclotron damping is treated as a perturbation on an appropriate complex Budden equation. The result agrees fairly well with the numerical solution in four test cases. The approximation for μ in R (23) could probably be improved to yield better results. This together with previous order-reduced descriptions[6–9], give approximate closed form expressions for all of the scattering coefficients in ion cyclotron heating for all heating scenerios.

ACKNOWLEDGEMENTS

This work was supported in part by the U.S. Department of Energy Contract No. DE-AC02-78ET-51013 and in part by the National Science Foundation Grant No. ECS-88-2475. The CCFM is supported by Hydro-Québec, Atomic Energy of Canada Ltd, and INRS-Energié.

APPENDIX: DERIVATION OF THE FAST WAVE EQUATION

We start with the standard zero-electron-mass, Vlasov-Maxwell local dispersion relation for ICRF where the dielectric tensor elements have been expanded to first order in $(k_\perp \rho_i)^2$. We neglect electron Landau damping and transit time damping. In the coupling region we assume slab geometry in which the tokamak toroidal magnetic field is directed along the z coordinate, and its gradient along x. The gradient scale length is R_0, the tokamak major radius. The dispersion relation normalized to the majority Alfvén velocity c_A is then

$$N_\perp^4 + N_\perp^2(K_0 - 2\lambda_N K_1) - 2\lambda_N K_0 + \lambda_N^2 = 0, \tag{A1}$$

where $c_A^2 = B_0^2/\mu_0 n_1 m_1$, $N_\perp = c_A k_\perp/\omega$, K_0 and K_1 are functions of x derived from the expanded dielectric elements and λ_N is a parameter depending on $N_\parallel$. In the case of D(H) for example

$$K_0 = -(\frac{1}{3} + \frac{\eta}{8} + N_\parallel^2) + \frac{\eta}{4N_\parallel\sqrt{\beta_H}} Z(\frac{\xi}{a_H}), \tag{A2}$$

$$K_1 = \frac{\sqrt{\beta_D}}{4N_\parallel} Z(\frac{\xi}{a_D}), \tag{A3}$$

where all of the symbols are the same as those defined in Section III. If the limit of large ξ is taken in (A1) we recover equation (3)

$$N_\perp^2 \equiv N_c^2 = \frac{2\lambda_N \tau + \lambda_N^2}{\tau}, \tag{A3}$$

where $-\tau$ is the constant terms of K_0. The dispersion relation (A1) can be rewritten in the form

$$N_\perp^2 = 2\lambda_N - \frac{\lambda_N^2}{K_0 + K_1 N_\perp^2}. \tag{A4}$$

The fast wave approximation amounts to substituing N_c for $N_\perp$ in the right hand side of (A4). The justification for this approximation is given in refs. 1 and 2. The basic idea is that $K_0 + K_1 N_\perp^2$ represents the IBW and it perturbs the FAW locally at $N_\perp = N_c$. If we make the substitution $-id/d\xi$ for $N_\perp$ in (A4) we get the fast wave equation (1). To recover the form of (1) we let $K_0 + K_1 N_c^2 = -\tau + \Gamma$, where Γ contains all of the ξ dependence in the Z functions. Extracting N_c^2 from the left hand side of (A4) gives the form of (1).

REFERENCES

1. D. Smithe, P. Colestock, T. Kammash, and R. Kashuba, Phys. Rev. Lett. **60**, 801 (1988).
2. P.L. Colestock and R.J. Kashuba, Nucl. Fusion **23**, 763 (1983).
3. E.F. Jaeger, D.B. Batchelor, and H. Weitzner, Nucl. Fusion **28**, 53 (1988).
4. H. Romero and J. Scharer, Nucl. Fusion **23**, 363 (1987).
5. M. Brambilla, Plasma Phys. Controlled Fusion **31**,723 (1989).
6. G. Francis, Ph.D. Thesis, Dept. of Physics, Massachusetts Institute of Technology, 1987.
7. V. Fuchs and A. Bers, Phys. Fluids **31**, 3702 (1988).
8. C. Lashmore-Davis, V. Fuchs, G. Francis, A.K. Ram, A. Bers, and L. Gauthier, Phys. Fluids **31**, 1641 (1988).
9. C. Chow, V. Fuchs, A. Bers, Analytical Studies of ICRF Heating, *Proceedings of the Eighth Topical Conference on Radio-Frequency Power in Plasmas, May 1-3, 1989, Irvine, Ca* (A.I.P. Conf. Proc. **190**, New York, 1989), p. 234.
10. H. Ye and A.N. Kaufman, Phys. Rev. Lett. **61**, 2762 (1988).
11. R.B. White and F.F. Chen, Plasma Physics **16**, 565 (1974).

FIGURE CAPTIONS

Figure 1: A sketch of the real part (solid line) and the imaginary part (dashed line) of the function $q^2(\xi)$ of the fast wave equation (1).

Figure 2: The approximate closed form solution (solid line) and the numerical solution (dashed line) of the reflection coefficient R plotted against $k_{\|}$ for CIT case 1. See the text for the parameters.

Figure 3: The approximate closed form solution (solid line) and the numerical solution (dashed line) of the reflection coefficient R plotted against $k_{\|}$ for CIT case 2. See the text for the parameters.

Figure 4: The approximate closed form solution (solid line) and the numerical solution (dashed line) of the reflection coefficient R plotted against $k_{\|}$ for Alcator C-mod. See the text for the parameters.

Figure 5: The approximate closed form solution (solid line) and the numerical solution (dashed line) of the reflection coefficient R plotted against $k_{\|}$ for PLT. See the text for the parameters.

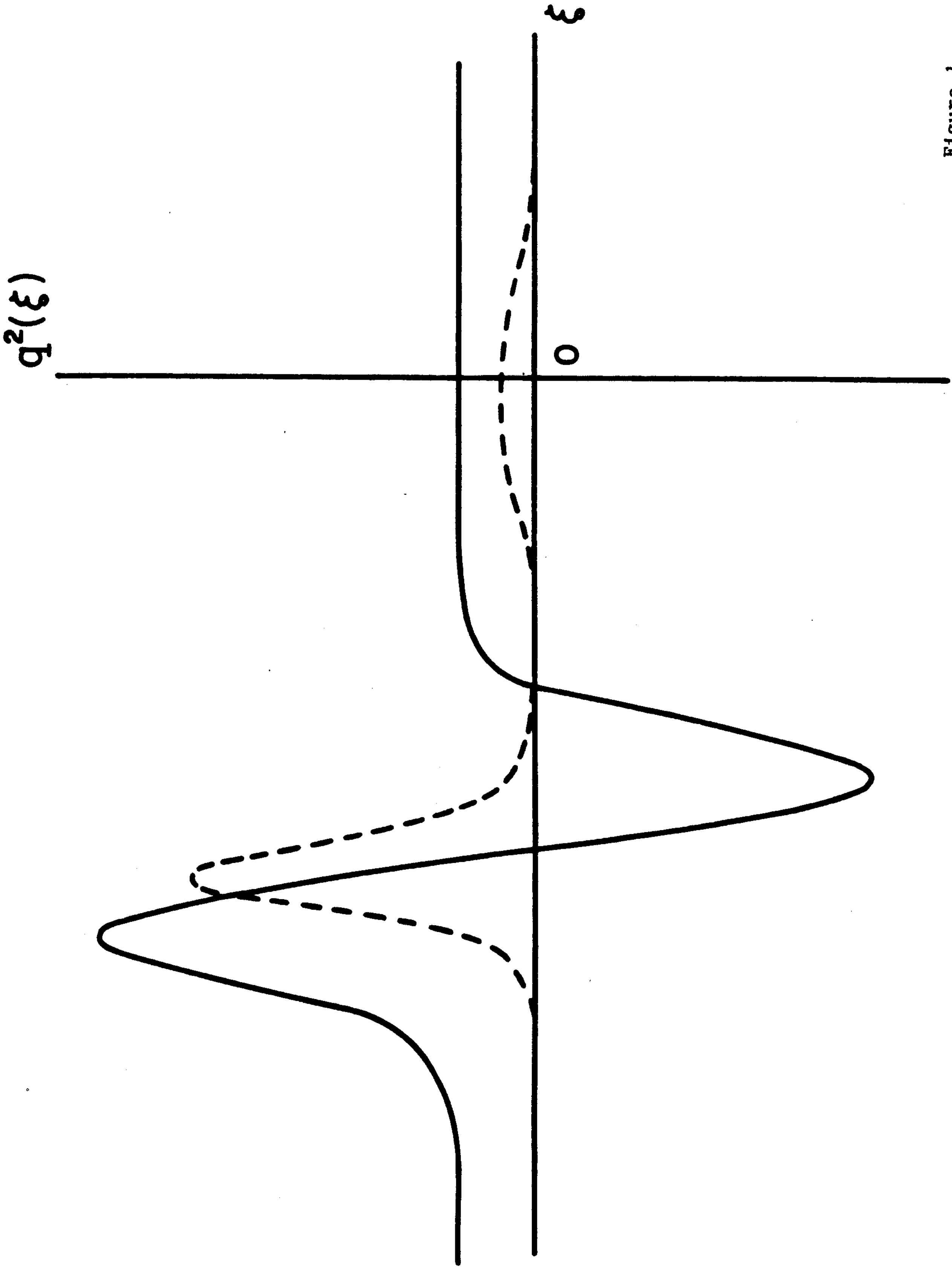


Figure 1

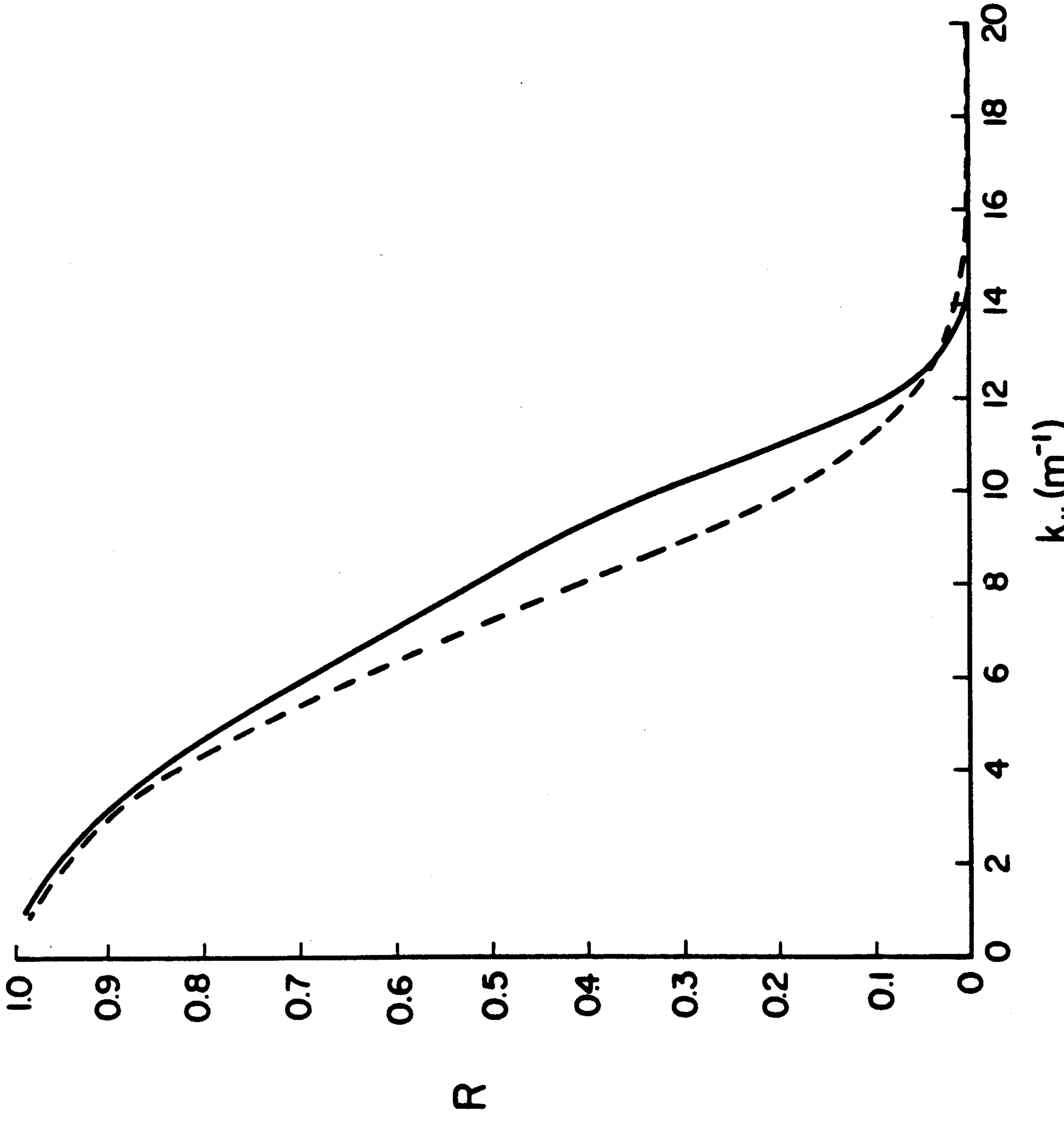
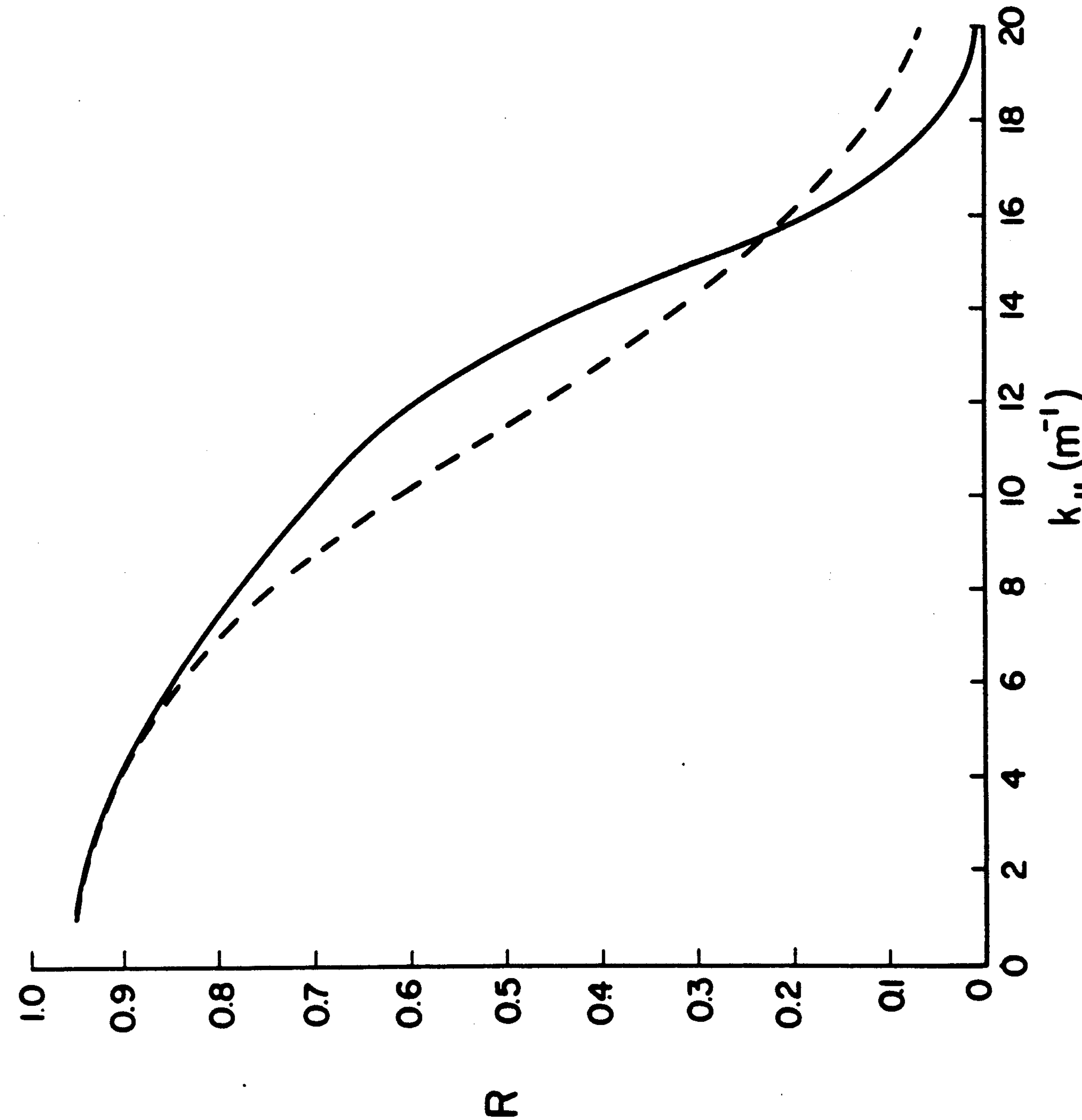


Figure 2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1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
14
16
18
20
$k_{\parallel}$ (m^{-1})

Figure 3

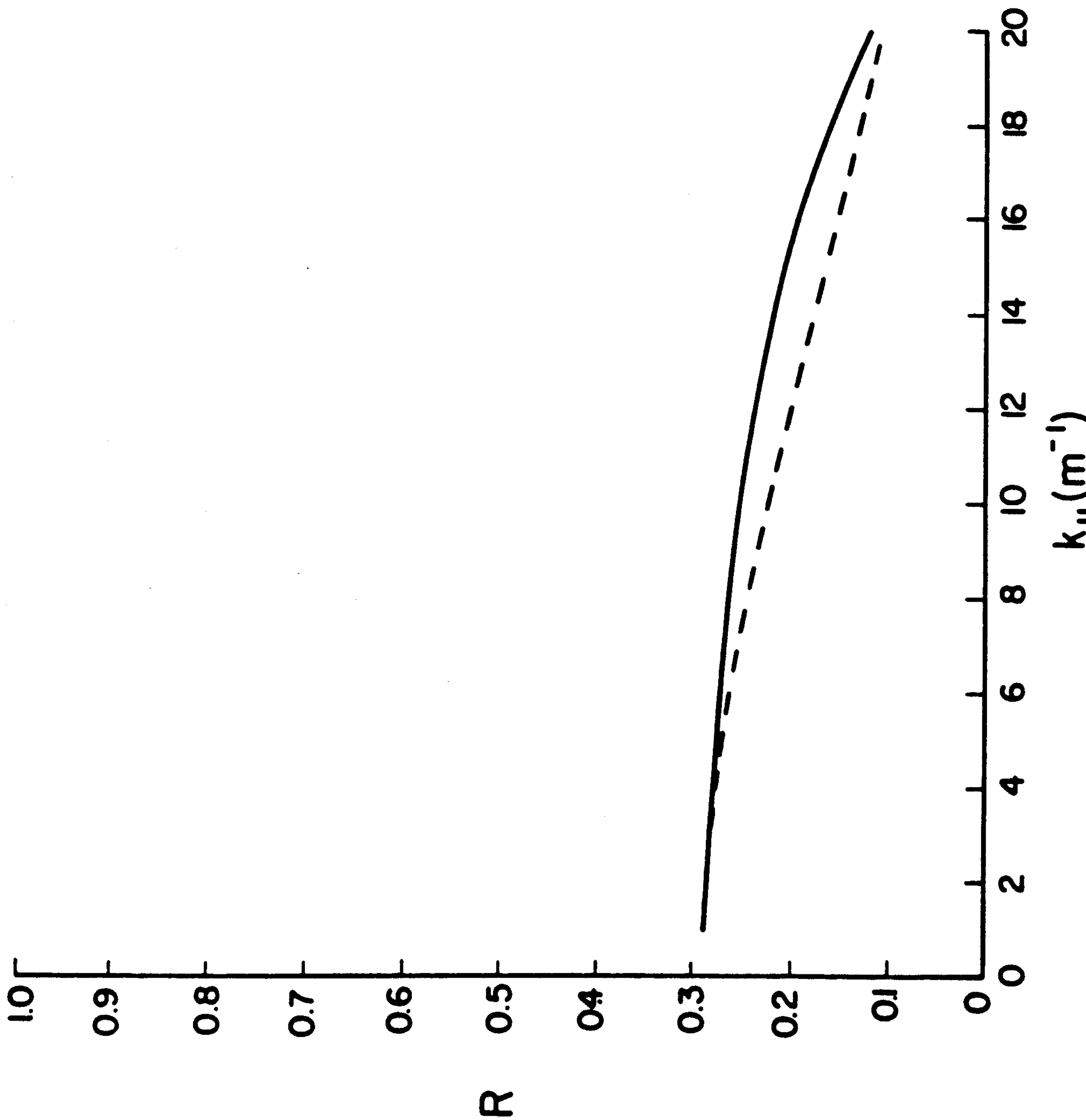

Figure 4

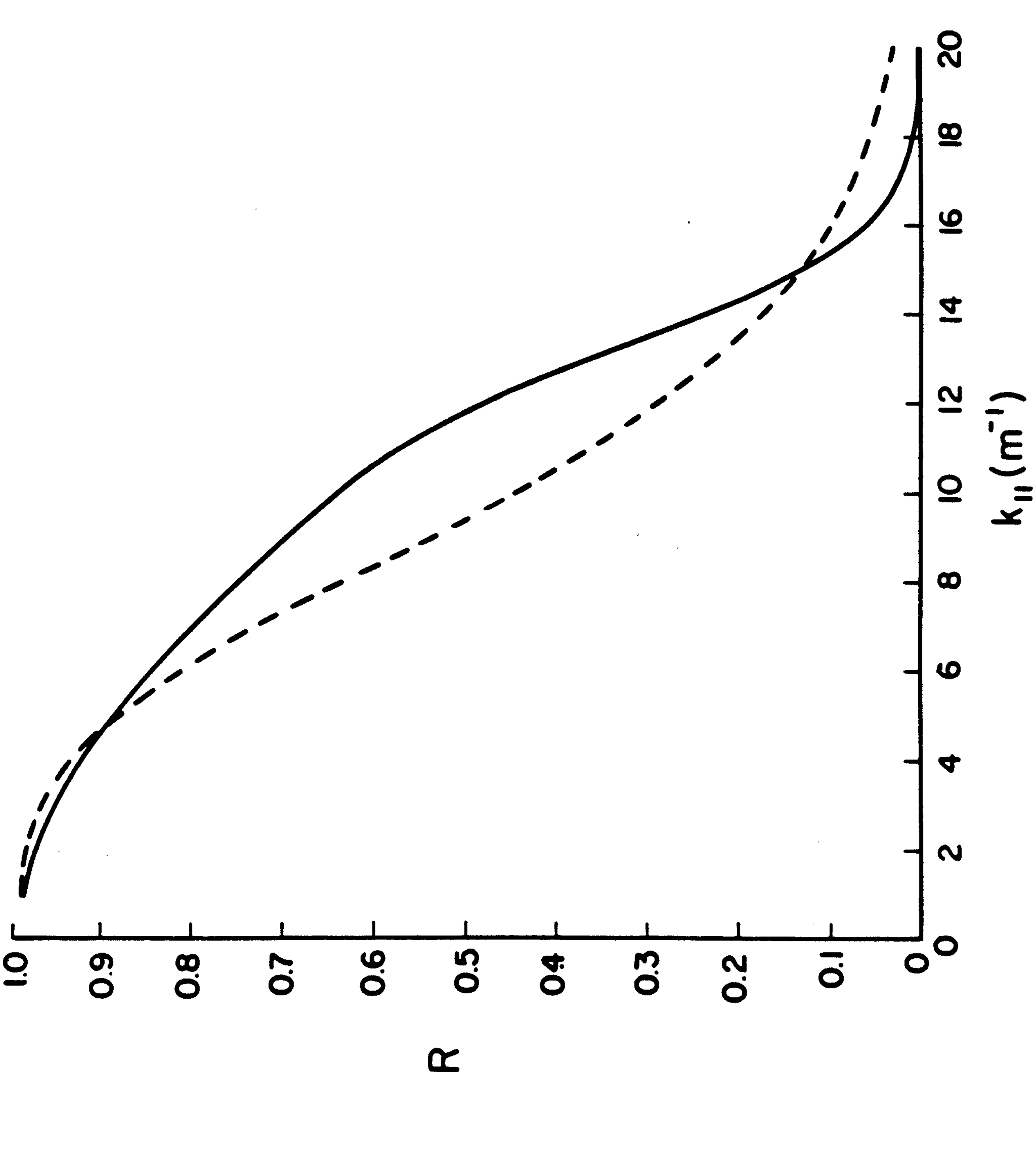
R
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
14
16
18
20
$k_{\parallel}(m^{-1})$

Figure 5